II CHAOS, PROBABILITY, AND THE COMPREHENSIBILITY OF THE WORLD

INTRODUCTION: COMPRESSIBILITY AND COMPREHENSIBILITY

There are quotations that mark important steps in the history of human thought. One of them is certainly this passage from Einstein:

> The very fact that the totality of our sense experiences is such that by means of thinking (operations with concepts, and the creation and use of definite functional relations between them, and the coordination of sense experiences to these concepts) it can be put in order, this fact is one which we shall never understand. One may say "the eternal mystery of the world is its comprehensibility."[1]

This mystery is seminally present in our prescientific cognition, but it reveals itself in full light only when one contemplates, as it has been expressed by Wigner, "the unreasonable effectiveness of mathematics in the natural sciences."[2]

What is meant here by the *effectiveness* of mathematics in the natural sciences is rather obvious (at least for those of us who are accustomed to the methods of modern physics). We model the world in terms of mathematical structures, and there exists an admirable resonance between these structures and the structure of the world. By means of experimental results the world responds to questions formulated in the language of mathematics. But why is this strategy *unreasonable*? In constructing mathematical theories of the world, we invest into them information we have gained with the help of the joint effort of former experiments and theories. However, our theoretical structures give us back more information than has been put into them. It looks as if our mathematical theories were not only information-processing machines, but also information-creating devices.

Let us consider an outstanding example. In 1915, after a long

1. A. Einstein, "Physics and Reality," chapter in *Ideas and Opinions,* New York: Dell, 1978, 283–315.

2. E. Wigner, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," *Communications in Pure and Applied Mathematics* 13 (1960) 1–14.

period of struggle and defeat, Einstein finally wrote down his gravitational field equations. He succeeded in deducing from them three, seemingly insignificant effects by which his general theory of relativity differed from the commonly accepted Newtonian theory of gravity. These effects were so small that the majority of physicists at that time could see no reason to accept a theory that required such a huge mathematical structure and yet explained so little. However, "the equations are wiser than those who invented them."[3] This is certainly true as far as Einstein's equations are concerned. In about half a century, physicists and mathematicians found a host of new solutions to these equations. Some represent neutron stars, gravitational waves, cosmic strings, stationary and rotating black holes, and so on. Fifty years ago nobody would even have suspected the existence of such objects. Now some of them have been discovered in the Universe,[4] and our confidence in Einstein's equations has grown so much that we are sure that the existence of at least some others will soon be experimentally verified. New information seems not only to be created by the mathematical equations, but surprisingly often it also corresponds well to what we observe if we focus our instruments on domains suggested by the equations themselves. It looks as if the structure of Einstein's equations somehow reflected the structure of the world: information about various strata of the world's structure seems to be encoded in the equations. By finding their correct solution and correlating it, through suitable initial or boundary conditions, with the given stratum of the structure of the world, we are able to decipher this information, and it often happens that the information was unavailable before we solved the equations.

We often read in philosophy of science textbooks that the mathematical description of the world is possible owing to *idealizations* made in the process of constructing our theories (we neglect the air resistance, the medium viscosity, or we invent nonexisting motions along straight lines, under the influence of no forces, and so on). This is a typical half-truth. At least in many instances, it seems that the idealization strategy does not consist in putting some information aside, but instead it is one of the most powerful mechanisms of the creation of information. For instance, the law of inertia (uniform motion under the influence of no forces!) has led us into the heart of classical mechanics. We should also notice that there were not experimental results that suggested which "influences" should be neglected, but it was the form of the equations of motion that selected those aspects of the world upon which the experiments should focus. The quantum world would remain closed to us forever if not for

3. This saying is ascribed to Hertz.

4. In this chapter, "Universe" will be used to refer to the Universe in the maximal possible sense. Later we will see that the Universe may contain many universes.

our mathematical models and idealizations on which they are based. Here we had no possibility at all to choose what should be taken into account and what should be left aside. We were totally at the mercy of mathematical structures. Almost all of the more important concepts of our everyday experience—such as localization, motion, causality, trajectory in space and time, individuality—drastically change their meanings when we move from the macroscopic world to the quantum world of elementary interactions. The only way to visualize what happens in this world is to enforce our imagination to follow mathematical structures and surrender to their explicative power.

Mathematics, as employed to reconstruct physical situations, enjoys another "unreasonable" property—it has enormous unifying power. In an almost miraculous way it unifies facts, concepts, models, and theories far distant from each other. The huge field of phenomena, investigated by contemporary physics, has been divided into a few subdomains, with each subdomain governed by a single equation (or system of equations). The equations of Einstein, Schrödinger, and Dirac are the best known representatives of this aristocratic family of equations. One printed page would be enough to write down the entirety of physics in a compressed form. We prefer fat volumes because we want to explore the architecture of these mathematical structures. We gain understanding by analyzing, step by step, the system of inferences, and by interpreting the formal symbols, triggering this subtle resonance between the logical structure and the results of measurement. We feel entitled to believe that these subdomains of physics, which until now were separated from each other, are but different aspects of the same mathematical structure. Although still remaining to be discovered, it is often credited with the name "Theory of Everything."

To express a law of physics in the form of a differential equation means to collect a potentially infinite set of events into a single scheme, in the framework of which every event, by being related to all other events, acquires significance and is explained. This is an example of what is called the *algorithmic compressibility.* However, what is really important is that it is always possible to disentangle what has been compressed. In the case of need, each event can be extracted from the entirety (but already in its reprocessed, significant form) by finding a suitable solution and choosing the corresponding boundary conditions. Today Einstein's question "Why is the world so comprehensible?" is very often formulated: "Why is the world algorithmically compressible?"[5] Indeed,

5. I will not discuss here the question of whether the statements "The world is comprehensible," and "The world is algorithmically compressible" are equivalent, or whether the latter is but a part of the former.

> without the development of algorithmic compressions of data all science would be replaced by mindless stamp collecting—the indiscriminate accumulation of every available fact. . . . Science is predicated upon the belief that the Universe is algorithmically compressible and the modern search for a Theory of Everything is the ultimate expression of that belief, a belief that there is an abbreviated representation of the logic behind the Universe's properties that can be written down in finite form by human beings.[6]

All these properties of mathematics, when applied to physical theories, often evoke in scientists the feeling of encountering something that is extremely beautiful. One could ask: Is mathematics beautiful because it is effective? This would be a utilitarian theory of beauty. In the more Platonic vein, one could ask: Do only beautiful mathematical structures prove to be effective in physics? Probably, these questions have no straightforward answers, but the fact that they are so often asked points toward the significant (albeit not yet sufficiently acknowledged) role of esthetics in the philosophy of science.

Was Einstein right when he was expressing his belief that the comprehensibility of the world will remain its "eternal mystery"? There is an attempt to neutralize Einstein's puzzlement over the question of why the world is so comprehensible by reducing all regularities present in the Universe to the blind game of chance and probability.

> It is just possible that complete anarchy may be the only real law of nature. People have even debated that the presence of symmetry in Nature is an illusion, that the rules, governing which symmetries nature displays, may have a purely random origin. Some preliminary investigations suggest that even if the choice is random among all the allowable ways nature could behave, orderly physics can still result with all the appearances of symmetry.[7]

Two essentially different implementations of this philosophy have been envisaged. The first, less radical, is an attempt that seeks to explain all regularities observed in the present Universe by reducing them to the chaotic (i.e., "most probable") initial conditions. The second, maximalistic one claims that the only fundamental law is the "game of probabilities," and all the so-called natural laws are but averages that won in this game. Although only partial results have been so far obtained in both of these approaches, the philosophical ideas lying behind them seem to be an interesting counterproposal with respect to Ein-

6. J. D. Barrow, *Theories of Everything: The Quest for Ultimate Explanation,* Oxford: Clarendon Press, 1991, 11. See also Joseph Ford's essay, "What is chaos, that we should be mindful of it?," in *The New Physics,* ed. P. Davies, New York: Cambridge University Press, 1989, 348–372, for clarification regarding the belief that the Universe is algorithmically compressible.

7. J. D. Barrow and J. Silk, *The Left Hand of Creation: The Origin and Evolution of the Expanding Universe,* London: Unwin, 1983, 213.

stein's philosophy, and certainly are worthwhile to discuss. This is the goal of this chapter.

The problem is of key importance for the topic of this chapter. In its most fundamental sense, God's action in the world consists in giving to the world its existence and giving it in such a way that everything that participates in existence also participates in its rationality, that is, is subject to mathematically expressible laws of nature. If Einstein's "mystery of comprehensibility" is indeed neutralized by the "pure game of chance and probability," then the central meaning of God's action in the world seems to be in jeopardy; anarchy takes over, and the world at its foundations is not rational. Because rationality and existence are very close to each other, the existence of the world, in turn, no longer seems to be the most profound locus of God's action but a random outcome of a degraded mystery.

Hence, I will show that such an attempt to neutralize Einstein's fascination with the comprehensibility of the world leads us even deeper into the mystery. Probability calculus is as good as any other mathematical theory; and even if chance and probability lie at the core of everything, the important philosophical and theological problem remains of why the world is *probabilistically comprehensible.* Why has God chosen probability as God's main strategy? In fact, the theory of probability permeates all aspects of our present understanding of the world. In particular, deterministic chaos theory and the theories of complexity and self-organization work because the world enjoys certain probabilistic properties.

We begin by presenting in more detail those approaches that attempt to explain the present world's regularities probabilistically. In section 2, we turn to the question of whether the chaotic initial conditions for the Universe are able to explain its actual structure. In section 3, we discuss the program of reducing all physical laws and symmetries to pure chance and randomness. To deal responsibly with the problems posed in the two preceding sections we must undertake a thorough discussion of the foundations of the probability calculus. This is the aim of section 4. The conclusions are drawn, and their theological implications are discussed in section 5.

"THE SHARPEST NEEDLE STANDING UPRIGHT ON ITS POINT"

Although the need to justify some large-scale properties of the observed Universe (such as its spatial homogeneity and isotropy) was noticed rather early by many authors, it was the paradigm of the anthropic principle that stressed the fact that the initial conditions for the Universe had to be extremely "fine tuned" to produce a world that could be subject for exploration by a living observer. There is no need to repeat here all arguments that have been quoted on behalf of this thesis.[8] All these arguments point to the fact that the present state of the Universe is as hard to produce from random initial conditions as it is hard to make "the sharpest Needle stand Upright on its Point upon a Looking-Glass" (this is Newton's expression ushered, essentially, in the same context).[9]

The additional difficulty is that each mechanism proposed to explain the large-scale properties of the Universe must first be able to overcome the barriers created by the existence of the limiting velocity of the propagation of physical interactions (the so-called horizon problem). Within the standard world model, to answer the question why a certain property of the Universe (e.g., the temperature of the microwave background radiation) is the same in regions that were never able to communicate with each other, one essentially needs to postulate the fine tuning of the initial conditions responsible for this fact.

An early proposal to overcome these difficulties goes back to C. W. Misner's classical works[10] on the so-called Mixmaster program in cosmology, nowadays more often known under the name of the chaotic cosmology. The idea is that it was the "mixing" character of physical processes in the very young Universe that led its large-scale properties to their present shape, independent of any initial conditions. To put it briefly, the "mixing" processes end up always producing identical universes regardless of their initial state. Various processes were tried as mixing candidates—hadron collisions, particle creation, neutrino viscosity—but all these mechanisms are strongly constrained by the horizon problem. This means that they can work efficiently only in those cosmological models that enjoy a very special geometric property: the expansion rate of the Universe must be related to the velocity of propagation of the mixing process such that the mixing should be able to reach distant regions of the Universe before they are too distant to be affected by them (the goalpost cannot recede faster than the runner can run). This means that there must exist a large-scale property of the Universe that controls the mixing process by synchronizing it with the global expansion rate and does this without exchanging physical signals between distant parts of the Universe. But this is exactly what we wanted

8. See J. D. Barrow and F. J. Tipler, *The Anthropic Cosmological Principle,* Oxford: Clarendon Press, 1986.

9. See the quotation in Chapter 9.2.

10. C. W. Misner, "The Isotropy of the Universe," *Astrophysical Journal* 151 (1968) 431–457; "Mixmaster Universe," *Physical Review Letters* 22 (1969) 1071–1074.

to avoid.[11] Let us note that this problem is strictly connected with the phenomenon of deterministic chaos. In fact, the Misner program does not work because—as deterministic chaos theory predicts—the relaxation time in the Mixmaster model is reached after an infinite lapse of time (the so-called Omega time appearing in Misner's equations).

The newer attempt to solve these difficulties, proposed by A. Guth[12] and A. D. Linde,[13] is known as the "inflationary scenario." In its standard version, at the epoch when the Universe was 10^{-35} seconds old, the splitting of the strong nuclear force from the electroweak force made the factor driving the world's evolution negative, and caused a rapid (exponential) expansion of the Universe, to be superimposed on its ordinary expansion. In the fraction of a second the radius of the Universe increased from about 10^{-23} cm to 10 cm (22 orders of magnitude!); that is, from something that was 10 billion times smaller than the size of a proton to something about the size of an orange. After this dramatic inflation phase, the Universe came back to its standard, much slower expansion. Such a rapid inflation erased from the Universe all vestiges of its pre-inflationary state; in this way, the initial conditions are unimportant. On the other hand, regions of the Universe, now very distant from each other, remember information from the epoch when they were in mutual contact. In this way the horizon problem can be overcome.[14]

However, one should notice that the inflationary strategy is able to explain probabilistically the present large-scale properties of the Universe only if the set of initial conditions leading to the inflationary phase is "large enough" in the space of all initial conditions. There are strong suspicions that this is not the case.[15] If this is true, we again face the problem of fine tuning to explain the inflation itself. And so, difficult questions return through the back door.

LAWS FROM NO-LAWS

The standard view today, underlying all efforts to achieve the final unification of physics, is that at extremely high energies, somewhere beyond the

11. See Z. Golda, M. Szydlowski, and M. Heller, "Generic and Nongeneric World Models," *General Relativity and Gravitation* 19 (1987) 707–718; A. Woszczyna and M. Heller, "Is a Horizon-free Cosmology Possible?" *General Relativity and Gravitation* 22 (1990) 1367–1386.

12. A. Guth, "Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems," *Physical Review* D23 (1981) 347–356.

13. A. D. Linde, "A New Inflationary Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems," *Physical Letters* 108B (1982) 389–393.

14. See *Inflationary Cosmology,* ed. L. F. Abbot and So-Young Pi, Singapore: World Scientific, 1986.

15. See G. F. R. Ellis and W. Stoeger, "Horizons in Inflationary Universes," *Classical and Quantum Gravity* 5 (1988) 207–220.

Planck threshold, everything had been maximally symmetric, and that subsequent breaking of this primordial symmetry led the Universe to its present diversified richness of forms. There is, however, another possibility: there could be no symmetry at high energies at all, or, equivalently, all possible symmetries could coexist on an equal footing, with order and law emerging only later from the primordial chaos. Barrow and Tipler, considering this possibility, ask: "Are there any laws of Nature at all? Perhaps complete microscopic anarchy is the only law of Nature."[16] What we now call the laws of nature would be the result of purely statistical effects, a sort of asymptotic state after a long period of averaging and selecting processes.

> It is possible that the rules we now perceive governing the behavior of matter and radiation have a purely random origin, and even gauge invariance may be an "illusion": a selection effect of the low-energy world we necessarily inhabit.[17]

There are several attempts to implement this philosophy in working physical models. I will mention two of them.

Within the so-called *chaotic gauge* program only preliminary results have been obtained so far. The idea is to show that physical laws and symmetries should arise, by some averaging processes, from a fundamental, essentially lawless and nonsymmetric level. In this approach, at low energies (i.e., on our macroscopic scale) one sees maximum symmetry, but this gradually disappears if we penetrate into more fundamental levels of high energies.[18] In particular, this approach should refer to gauge symmetries, which seem to play an ever-increasing role in contemporary physics. The proponents of this program write: "It would be nice to show that gauge invariance has a high chance of arising spontaneously even if nature is not gauge invariant at the fundamental scale."[19] Or more technically, "it would be nice" to show that if the Lagrangian, from which physical laws are to be derived, is chosen at random, then at low energies local gauge invariance will emerge, and it will be a stable property in the space of all possible Lagrangian-based theories. However, the same authors were able to show that only a gauge theory arises at low energies from a theory that at high energies differs from the exactly gauge invariant theory by no more than a specified amount of noninvariant interactions; that is, that it is enough to assume an approximately gauge invariant theory at high energies to obtain the usual gauge theory on our scale. Advocates of this program express

16. J. D. Barrow and F. Tipler, *The Anthropic Cosmological Principle,* 256.

17. Ibid.

18. See J. Iliopoulos, D. V. Nanopoulos, and T. N. Tomaras, "Infrared Stability or Anti-Grandunification," *Physics Letters* 94B (1980) 141–144.

19. D. Foerster, H. B. Nielsen, and M. Ninomiya, "Dynamical Stability of Local Gauge Symmetry," *Physics Letters* 94B (1980) 135–140.

their hope that, by using this strategy, it would be possible to estimate the order of magnitude of at least some fundamental constants and to demonstrate their quasi-statistical origin.

Another possibility is Linde's *chaotic inflationary cosmology.*[20] The dynamics of the Linde universe is dominated by a nonequilibrium initial distribution of a noninteracting scalar field φ with the mass m much less than the Planck mass $M_p \sim 10^{19}$ GeV. If the Universe contains at least one domain of the size $l \geq H^{-1}(\varphi)$ with $\varphi \geq M_p (M_p/m)^{1/2}$, where $H = R'/R$, R being the scale factor of the locally Friedman universe, it endlessly reproduces itself in the form of inflationary mini-universes. In fact, this reproduction process leads to an exponentially growing number of causally noninteracting universes. Because the birth of each new mini-universe is independent of the history of the mother universe, "the whole process can be considered as an infinite chain reaction of creation and self-reproduction which has no end and which may have no beginning."[21] When, during such a birth process, the Universe splits into many causally disconnected mini-universes of exponentially growing sizes, "all possible types of compactification and all possible vacuum states are realized."[22] This leads to various physics in various daughter-universes. Linde writes:

> When several years ago the dimensionality of spacetime, the vacuum energy density, the value of electric charge, the Yukawa couplings, etc., were regarded as true constants, it now becomes clear that these "constants" actually depend on the type of compactification and on the mechanism of symmetry breaking, which may be different in different domains of the universe.[23]

In this way, a "chaos"[24] is realized not within the one Universe but within the ensemble of many universes, and some sort of the anthropic principle is necessary if our "local Universe" is to have the physical laws we now discover and the structure we now observe.[25]

20. See A. D. Linde, *Fizika Elementarnykh Chasitis I Inflatsonnaia Kosmologiia* [Physics of Elementary Particles and Inflationary Cosmology], Moscow: Nauka, 1990, and by the same author, "Inflation and Quantum Cosmology," in *300 Years of Gravitation,* ed. S. W. Hawking and W. Israel, Cambridge: Cambridge University Press, 1987, 604–630.

21. A. D. Linde, "Inflation and Quantum Cosmology," 618.

22. Ibid., 627. Roughly speaking, by compactification Linde understands the process by which the number of space-time dimensions is established inside the newly-born mini-universe; this number may be different from that in the mother universe.

23. Ibid.

24. In this case, as in the case of a chaotic gauge program, the term "chaos" is not used in the technical sense of deterministic chaos, although one could expect that in both cases deterministically chaotic phenomena (in the technical sense) are involved.

25. See J. D. Barrow, *The Worlds Within the World,* Oxford: Clarendon Press, 1988, 281–289. Later on, Linde's hypothesis was modified and developed by Lee Smolin; see his book *The Life of the Cosmos,* Oxford: Oxford University Press, 1997. Critical remarks in the following sections refer to his ideas as well.

PROBABILISTIC COMPRESSIBILITY OF THE WORLD

The strategies presented in the two preceding sections were aimed at understanding the Universe by reducing its laws and structure to a pure game of probabilities. Our first reaction to such strategies is that if one of them succeeds (especially one of their stronger versions presented in section 4), then the "eternal mystery of the world's comprehensibility" that Einstein stressed would disappear: comprehensibility would give place to probability, and mystery would change into averaging mechanisms. However, to go beyond "first reactions" and to assess critically such an approach to the "rationality of the world," we must turn to the foundations of the probability calculus. This is the aim of this section.

Many branches of modern mathematics have their origin in an interplay of theory and application. This is also true as far as the probability calculus is concerned. Moreover, one would be inclined to say that in this case more depends on application than on theory. This is not only because the probability calculus originated from experience but mostly because it is very difficult to separate the very notion of probability from its empirical connotations. This fact gave rise to many philosophical discussions concerning the foundations of probability. In what follows I will try to avoid entering into these discussions; instead, I will trace the meaning of some fundamental concepts by placing them within the mathematical structure of the probability theory in its standard (Kolmogorov) formulation.[26]

In the contemporary standard approach, probability theory is a special instance of measure theory. *Measure,* in the mathematical sense, is a function defined on subsets of a certain space called the *measure space.* These subsets, called *measurable subsets,* can be thought of as objects to be measured. The function defined on these objects ascribes to each of them the result of a measurement (i.e., its measure, a number). For instance, the objects in question could be subsets of the Euclidean space, and the measure a function ascribing to each subset its volume. From the mathematical point of view, the essential circumstance is that outside the measure space the concept of measuring is meaningless.

Some cases are known in which not every subset of a given space is measurable. In such a space there are "things" (subsets) that cannot be measured, that

26. Regarding different views and philosophies of probability, see D. Home and M. A. B. Whitaker, "Ensemble Interpretations of Quantum Mechanics: A Modern Perspective," *Physics Reports* 210 (1992) 233–317.

is, no measurement result can be meaningfully ascribed to them. This runs counter to the common view that "what cannot be measured does not exist." Such subsets might indeed seem rather unusual, but one can find them even in the open interval (0,1) of real numbers.[27]

Probability is just a measure satisfying one additional condition: the measure of the entire space should be equal to one. Consequently, the measure of any of its subsets is either zero or a fraction between zero and one. If this axiom is satisfied the measure space with its measurable subsets is called *probability space,* and the measure defined on it the *probability distribution*.

Let us notice that so far there is nothing in our theory that would suggest an uncertainty or indeterminacy we intuitively connect with the idea of probability. All consequences follow from their axioms in a strictly apodictic manner, exactly the same as in mathematical theories. Intuitions that we connect with the concept of probability enter our theory via its reference to reality, that is, via its interpretation. The standard method of referring mathematics to reality is by the intermediary of physics. Some mathematical structures are used as building blocks of a physical theory, and the task of this theory is to investigate the world. The mathematical theory of probability, however, seems to relax this rule. It often makes references to reality with no direct help of a physical theory. For instance, when making probabilistic predictions of the outcome of throwing dice or of the price increase in an approaching fiscal year, a certain physical-like interpretation of a mathematical structure must intervene; but it is so natural and so closely linked to the mathematical structure itself that we prefer not to call it a physical theory but rather a *probabilistic model* of a given situation (with no reference to physics).

To be more precise, physical intuition enters the probabilistic model through the definition of the probability distribution. For instance, if we want to model playing with ideal dice mathematically, we define the probability distribution as a function that ascribes to each *elementary event*—that is, to each of six possible outcomes—the value of the probability measure equal to 1/6. This particular value is taken from experience, namely, from a long series of throwing dice, but once put into the definition of the probability distribution, it becomes a structural part of the mathematical theory itself.

The feeling of a "probabilistic uncertainty" is connected with the *frequency*

27. For example, let a and b be real numbers in the open interval (0,1). If $a–b$ is a rational number we write $a \# b$. This is clearly an equivalence relation. We define A to be a subset of real numbers consisting of exactly one number of each equivalence class. It can be shown that A is not measurable. See R. Geroch, *Mathematical Physics,* Chicago: University of Chicago Press, 1985, 254–255.

interpretation of the distribution function defined in the above way. The value 1/6 of the distribution function at a given (elementary) event—for instance at the event "outcome three"—is interpreted as giving the relative frequency of the "outcome three" (i.e., the ratio of the number of the fortuitous events, in our case "outcome three," to all possible events) in a long series of throwing dice. Indeed, such experiments show that, in this circumstance, relative frequencies are approximately equal to 1/6. The longer the series of throws, the closer the relative frequency approximates this value. This property of the world is known as its *frequency stability.* It is a property of the world and not the property of the mathematical theory, because it is taken from experience and has no justification in the theory itself.

The frequency stability of the world is of fundamental importance for our analysis. In both everyday life and physics, we often meet random events or random experimental results. The result of an experiment is said to be random if it is not uniquely determined by the conditions under which the experiment is carried out and which remain under the control of the experimenter. Subsequent results of such an experiment are unpredictable. If in a series of n such experiments, n_A experiments give the result A, and $n - n_A$ give some other results, the number $f(A) = n_A/n$ is called the *frequency* of A. It turns out that as n is larger and larger, $f(A)$ approaches a certain number more and more closely. This tendency to certain numerical results reflects the world's frequency stability.

This is indeed an astonishing property. One cannot see any a priori reason why the world should be stable in this respect. But the world is frequency stable, and it is clear that without this property the probability calculus could not be applied to analyze the occurrence of events in the world. We can say that owing to its frequency stability the world is *probabilistically compressible.* A priori we could expect that truly chaotic or random phenomena would evade any mathematical description, but in fact the description of phenomena we call random or chaotic is not only possible but can be compressed into the formulae of the probability theory. The probabilistic compressibility of the world turns out to be a special instance of its algorithmic compressibility, and one would dare to say that it is the most astonishing (or the most unreasonable) instance of it.

This is even more the case if we remember that the applicability of probabilistic ideas to the real world underlies much of the foundations of statistical physics, and also the derivation of the classical limit of quantum theory, as well as the analysis of observations. All these aspects of probability applications are closely related to the problem of the arrow of time. Because the laws of fundamental physics are time reversible they must be involved in a subtle game of probabilities in order to produce irreversible phenomena on a macroscopic scale. There are strong reasons to suspect that the answer to the question of why the cosmic process evolves in time, rather than being reduced to an instant, is but another aspect of the probabilistic compressibility of the world.

We should not forget that probability theory is as good as any other mathematical theory. The distribution function is defined by idealizing some experimental results, but the probabilistic model, once constructed, produces uniquely determined results. The frequency interpretation of the probabilistic axioms does not influence formal inferences or the manipulation of formulae; it only allows us to look at the Universe in a special way—in a way in which events are not just given but seem to have a certain potentiality to happen, and the cosmic process does not just unfold but seems to have the possibility of choosing various branches in this unfolding.

The above considerations have shown that even if we were able to reduce the comprehensibility of the world to its probabilistic compressibility (as it was presupposed by the strategies and philosophies presented in sections 2 and 3), the questions would remain: Why does the probability theory apply to our world? Why has our world the property of being frequency stable?

When we ask the question, "Why is the world mathematical?" we should also wonder why is it subject to the "game of probabilities." Clearly, the riddle of probability does not eliminate the mystery of comprehensibility.

GOD OF PROBABILITIES

The ideas presented in sections 2 and 3 have their origin in an interesting property of the human mind, for which the high probability of an event is a kind of sufficient reason for its occurrence, but low probabilities always call for some special justification. One could guess that this property of our mind has evolved through an intricate agglomeration of selection effects in the world, the structure of which is predominantly shaped by frequency stable processes.

In classical natural theology, the justification of low probability events was often sought in the direct action of God. The low probability itself was considered to be a gap in the natural course of events, a gap that had to be filled in by the "hypothesis of God." In this way, high probability becomes a rival of God. We hear the echo of such views in metaphors contemporary scientists sometimes evoke to impress the reader with how finely the initial conditions should be tuned to produce the Universe in which the reader-like being could be born and evolve. For instance, in the famous book by Roger Penrose, the caption under the picture of God pointing with the pin to the initial conditions (or equivalently to the point in the phase space) from which God intends to create the world, reads:

> In order to produce a universe resembling the one in which we live, the Creator would have to aim for an absurdly tiny volume of the phase space of possible universes—about the entire volume.[28]

On the contrary, the attempt to reduce phenomena to random events hiding behind them (e.g., to random initial conditions) is often thought of as supporting an atheistic explanation. For instance, the main argument of Leslie's book on the anthropic principles[29] is that the principal competitor of the God hypothesis is the idea of multiple worlds in which all possibilities are realized, along with some observational selection effects that would justify our existence as observers of the world. The God hypothesis relies on the argument from design, which is "based on the fact that our universe looks much as if designed." However, there might be immensely many universes.

> And their properties are thought of as very varied. Sooner or later, somewhere, one or more of them will have life-permitting properties. Our universe can indeed look as if designed. In reality, though, it may be merely the sort of thing to be expected sooner or later. Given sufficiently many years with a typewriter even a monkey would produce a sonnet.[30]

A different view on probability came with the advent of quantum mechanics. The Hilbert space, an arena (in fact, the phase space) on which quantum processes occur, is a very beautiful and very solid mathematical structure, but when interpreted in a standard probabilistic way it reveals the unexpected image of the microworld. Wave functions, containing all information about a quantum object, are essentially nonlocal entities; they are defined "everywhere": for instance, from the wave function you can compute the probability of finding an electron at any place in the Universe. Wave functions evolve in time in a strictly deterministic way, but when a measurement is performed, deterministic evolution breaks down, all available information reduces to the unique measurement result, an infinite number of possibilities collapse to the

28. R. Penrose, *The Emperor's New Mind: Concerning Computers, Minds, and the Laws of Physics,* New York: Oxford University Press, 1989, 343.

29. J. Leslie, *Universes,* London and New York: Routledge, 1990. See also, J. D. Barrow and F. J. Tipler, *The Anthropic Cosmological Principle.*

30. J. Leslie, *Universes*, 1. Leslie clearly expresses his own opinion: "While the Multiple Worlds (or World Ensemble) hypothesis is impressively strong, the God hypothesis is a viable alternative" (p. 1).

single eigenvalue of the measurement operator.[31] Less informed philosophers speak about the free will of electrons; better informed ones begin to see that the time-honored antinomy between lawfulness and probability should be reconsidered *ab initio.*

Recent developments in deterministic chaos theory have shown that this is also true as far as the macroscopic world is concerned. An instability of the initial conditions leads to unpredictable behavior at later times, and there are strong reasons to believe that a certain amount of such a randomness is indispensable for the emergence and evolution of organized structures.

The shift we have sketched in our views on the significance of probability has had its impact on modern natural theology. Randomness is no longer perceived as a competitor of God, but rather as a powerful tool in God's strategy of creating the world. For instance:

> God is responsible for ordering the world, not through direct action, but by providing various potentialities which the physical universe is then free to actualize. In this way, God does not compromise the essential openness and indeterminism of the universe, but is nevertheless in a position to encourage a trend toward good. Traces of this subtle and indirect influence may be discerned in the progressive nature of biological evolution, for example, and the tendency for the universe to self-organize into a richer variety of ever more complex forms.[32]

Or:

> On this view God acts to create the world through what we call "chance" operating within the created order, each stage of which constitutes the launching pad for the next. However, the actual course of this unfolding of the hidden potentialities of the world is not a once-for-all pre-determined path, for there are unpredictabilities in the actual systems and processes of the world (micro-events at the "Heisenberg" level and possibly non-linear dynamical complex systems). There is an open-endedness in the course of the world's "natural" history. We now have to conceive of God as involved in explorations of the many kinds of unfulfilled potentialities of the universe(s) he has created.[33]

Still, either a God with a sharply pointed pin in hand choosing the improbable initial conditions for the Universe, or a God exploring the field of possibil-

31. In quantum theory, any measurement is represented by an operator acting on the corresponding wave function. Eigenvalues of this operator represent possible results of the measurement.

32. P. Davies, *The Mind of God: The Scientific Basis for a Rational World,* New York: Simon and Schuster, 1992, 183. Davies refers here to Whitehead's philosophy of God.

33. A. R. Peacocke, "God as the Creator of the World of Science," in *Interpreting the Universe as Creation: A Dialogue of Science and Religion,* ed. V. Brümmer, Kampen, The Netherlands: Kok Pharos, 1991, 110–111.

ities by playing with chance and randomness, seems to be but a Demiurge constrained by both a chaotic primordial stuff and the mathematical laws of probability (just as Plato's Demiurge was bound by the preexisting matter and the unchanging world of ideas). Of course, we could simply identify the laws of probability with God (or with the ideas present in God's mind), but this would bring us back to all traditional disputes surrounding the Platonic interpretation of God and mathematics.

Instead of immersing ourselves in risky disputes, I believe we should once more ask Einstein's question: Why is the world so comprehensible? As we have seen, there is no escape from this question via the "game of probabilities," for if we reduce comprehensibility to probability, new questions will emerge: Why should the theory of probability be privileged among all other mathematical theories?[34] Why is the world probabilistically compressible? And if the answer to the last question is: The world is probabilistically compressible because it enjoys the property of being frequency stable, we will then ask: Why is it frequency stable?

Any natural theology is sentenced to the "God-of-the-gaps" strategy. But if there are no gaps in the natural order of things, if the world is a self-enclosed entity, then there is no way from the world to its maker. The essential point is to distinguish between spurious gaps and genuine ones. Spurious gaps are temporary holes in our knowledge usually referring to an incomplete scientific theory or hypothesis and to a restricted domain of phenomena. Genuine gaps are truly disastrous; they overwhelm everything. I think that all gaps are spurious except for the following two or three.

First is the *ontological gap.* Its meaning is encapsulated in the question: Why is there something rather than nothing? The problem at stake is sheer existence. Even if we had a unique theory of everything (and some physicists promise us we will have it in the not too distant future), the question would remain of who or what "has breathed fire into the equations" to change what is merely a formally consistent theory into one modeling the real universe.

Second is the *epistemological gap:* Why is the world comprehensible? I have dealt with this question in the present chapter. It is truly a gap. Science presupposes the intelligibility of the world but does not explain it. Philosophy of science can at most demonstrate the nontrivial character of this question, but remains helpless if one further asks, "Why?"

From the theological perspective, both gaps, the ontological gap and the epistemological one, coincide: everything that exists is rational, and only the

34. To see that this question is not trivial, see section 4.

rational is open for existence. The source of existence is the same as the source of rationality.

I strongly suspect that there is a third genuine gap; I would call it the *axiological gap*—it is connected with the meaning and value of everything that exists. If the Universe is somehow permeated with meaning and value, they are invisible to the scientific method, and in this sense they constitute the real gap as far as science and its philosophy are concerned. Here again, by adopting the theological perspective, I would guess that the axiological gap does not differ from the remaining two: the source of existence, rationality, and value is the same.

Modern developments in science have discovered two kinds of elements (in the Greek sense of this word) shaping the structure of the Universe—the *cosmic elements* (integrability, analycity, calculability, predictability) and the *chaotic elements* (probability, randomness, unpredictability, and various stochastic properties). I think I have convincingly argued in this chapter for a thesis that the chaotic elements are in fact as "mathematical" as the cosmic ones, and if the cosmic elements provoke the question of why the world is mathematical, the same is true as far as the chaotic elements are concerned. On this view, *cosmos* and *chaos* are not antagonistic forces but rather two components of the same Logos immanent in the structure of the Universe.[35] Einstein's question, "Why is the world so comprehensible?", is a deeply and still not fully understood theological question.

35. For examples of such a cooperation between "cosmic" and "chaotic" elements, see my paper: "The Non-Linear Universe: Creative Processes in the Universe" (especially section 5), in *The Emergence of Complexity in Mathematics, Physics, Chemistry, and Biology,* ed. B. Pullman, Pontificiae Academiae Scripta Varia, vol. 89, Vatican City, 1996, 191–209.